Modeling and Inference for Precipitation Time Series

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Summary

- Goal: Build a statistical model for precipitation time series
- Challenges:
  - Precipitation data have a large proportion of zeros
  - They contain extreme values
  - May exhibit autocorrelation
  - Building a statistical model with all the above features can be difficult
- We consider two approaches
  - Generalized Autoregressive Moving Average model (Zheng et al. 2015)
  - Weibull Model with Markov transitions (extending Wilks 1999)

Why Is This of Interest?

- Climate change will affect precipitation
- Some areas will experience more severe drought, others will have more extreme rainfall
- Understanding precipitation patterns and trends may be useful for public policy
- Useful for studying climate models

Data

- Precipitation data obtained from the NOAA Climate Data Online database
- Focus here on Warren, PA and Phoenix, AZ

GARMA Model

- Time series data: $Y_1, Y_2, \ldots, Y_T$
- $Y_t \sim \text{Gamma}(cY_{t-1}^{\varphi d}, cY_{t-1}^{\varphi(d-1)})$ where c, φ, d are parameters
- It uses a log -link function:
  - $\log(Y_t) = \varphi \log(Y_t) + e_t$
  - $e_1, e_2, \ldots, e_T$ are the error terms
- We allow for a seasonal effect via including a monthly term:
  - $\log(Y_t) = \mu_t + \varphi \log(Y_t) + e_t$
- We use Markov Chain Monte Carlo to estimate the parameters

Warren, PA Best Model: Gamma-GARMA(0,0)

- With monthly means regression
- Very little autocorrelation between months
- The monthly means model does a good job of modeling this structure

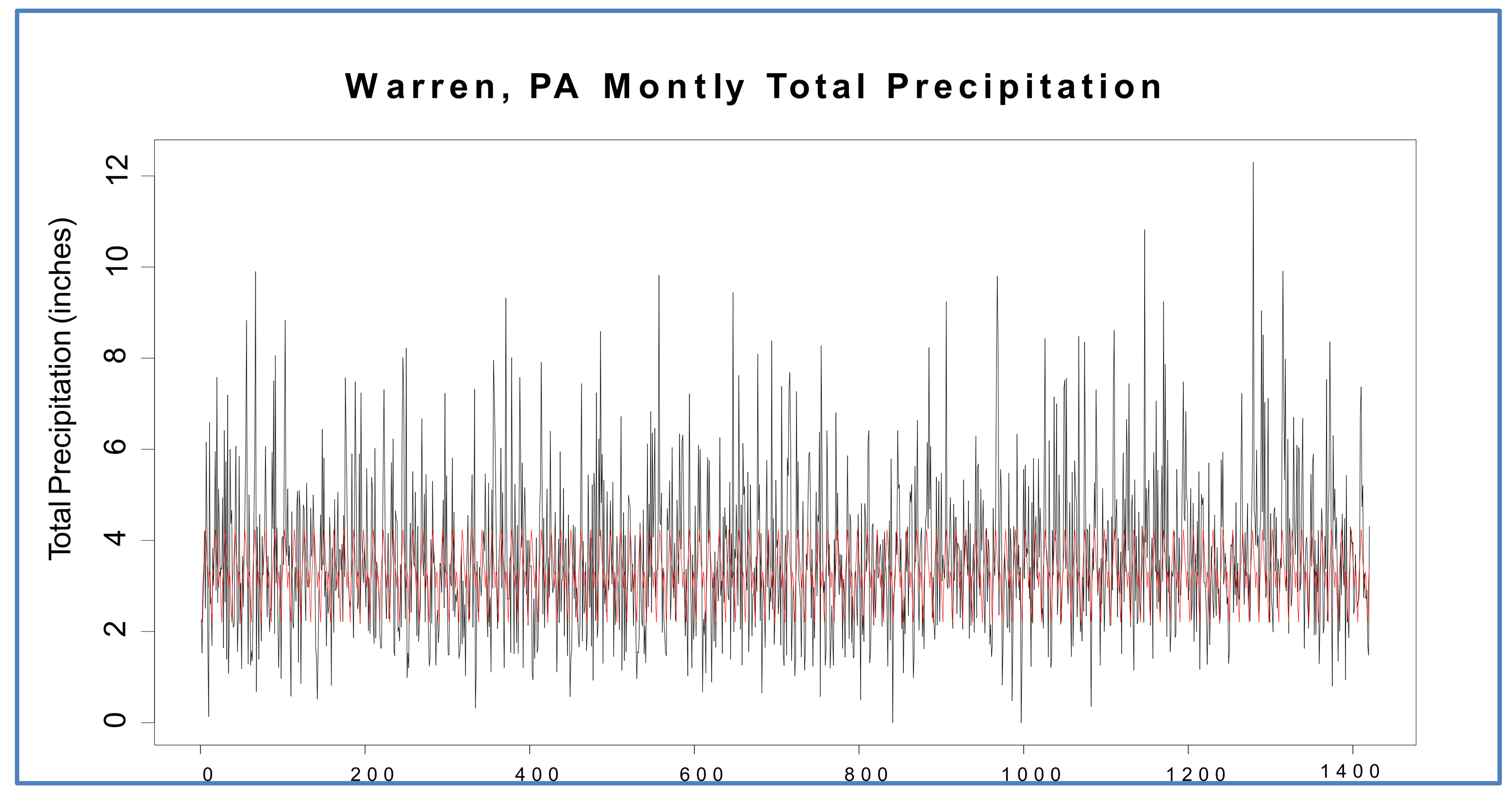

Figure 2 – Warren, PA (1897-2015)
The precipitation data is in black, with GARMA forecasts in red.

Phoenix, AZ Best Model: Gamma-GARMA(1,0)

- A monthly means regression model still has correlated residuals
- This model accounts for the autocorrelation that was seen in the data

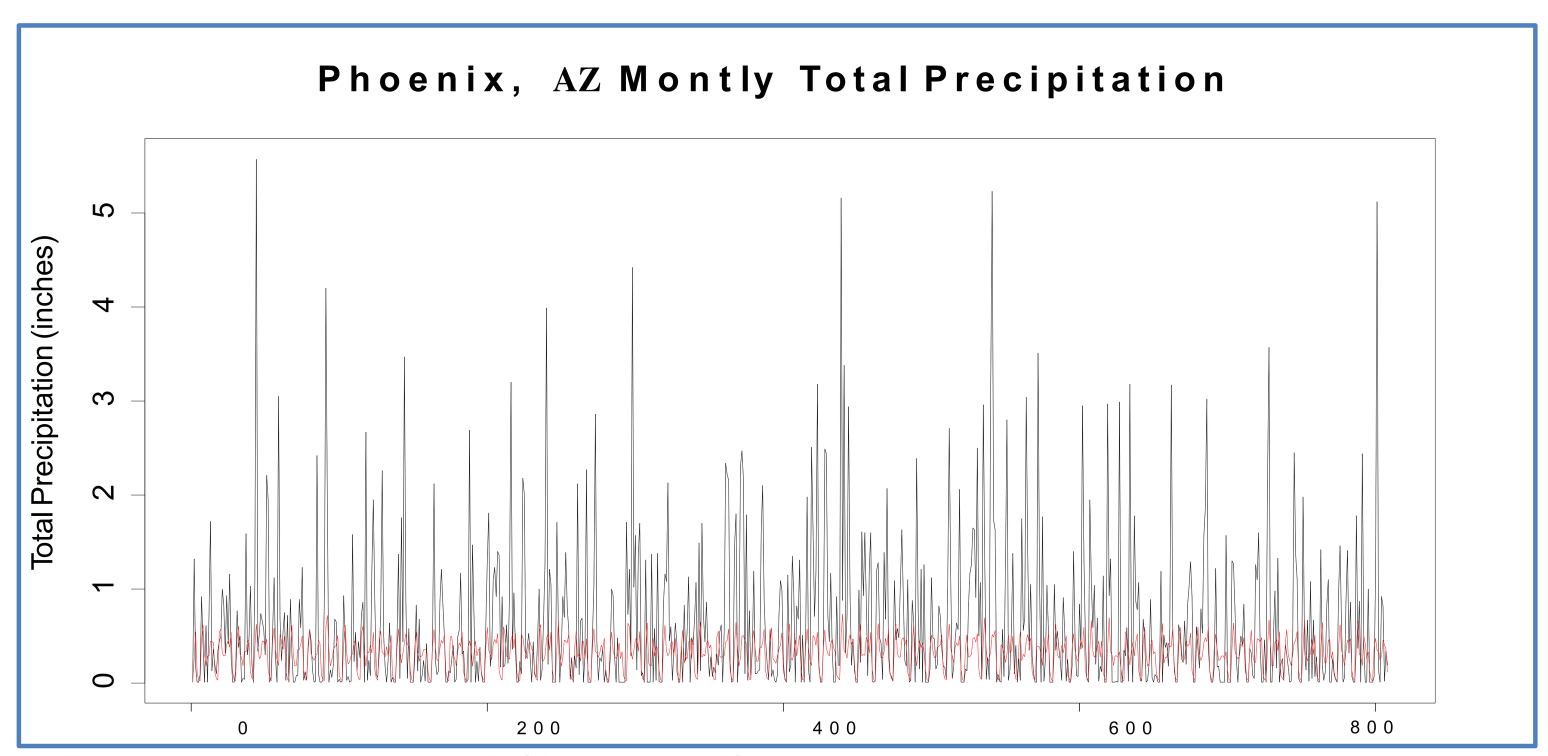

Figure 3 – Phoenix, AZ (1948-2015)
The precipitation data is in black, with GARMA forecasts in red.

GARMA Discussion

- The GARMA models do a good job at accounting for autocorrelation
- Fail to account for zeros in data and extreme values

Weibull Model

- $Y_t \sim \text{Weibull}(\lambda, k)$ with probability P^k_{ij}
- Where P^k_{ij} is the transition probability of a Markov chain, with i, j = {Dry, Wet}, and k={1,2,3,…,12}, corresponding to the months of the year
- Time dependence between presence-absence (rain/ no rain) via Markov model
- Amount of rain is conditionally independent given rain/ no rain:
  - If it rains, amount is drawn independently

Warren, PA

- Weibull model accounts for monthly variability of the amount of rain on wet days

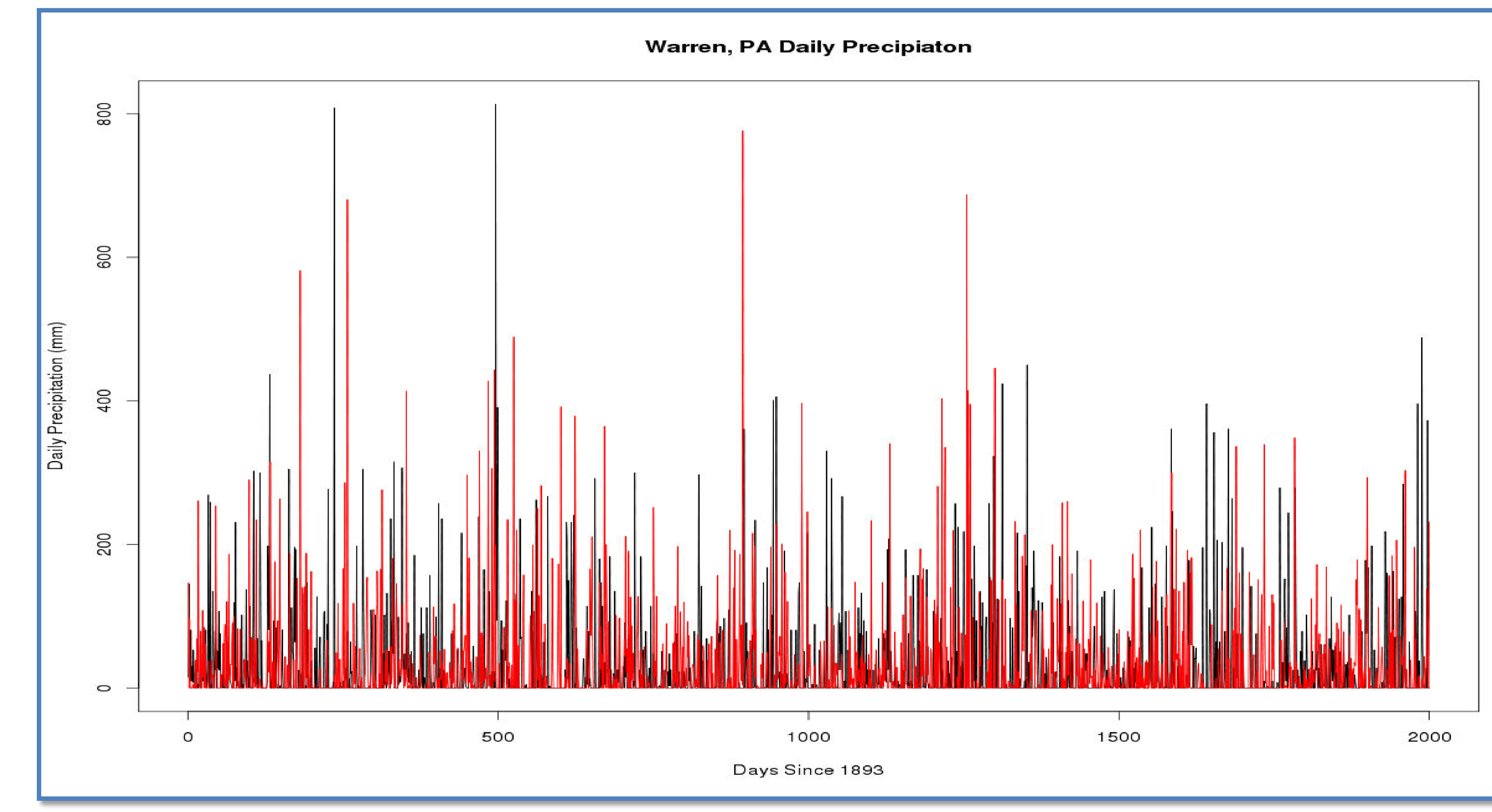

Figure 3 – Warren, PA since 1893
The precipitation data is in black, and a Weibull simulation in red.

Phoenix, AZ

- Markov model accounts for variability throughout the year of transitions between rain/ no rain

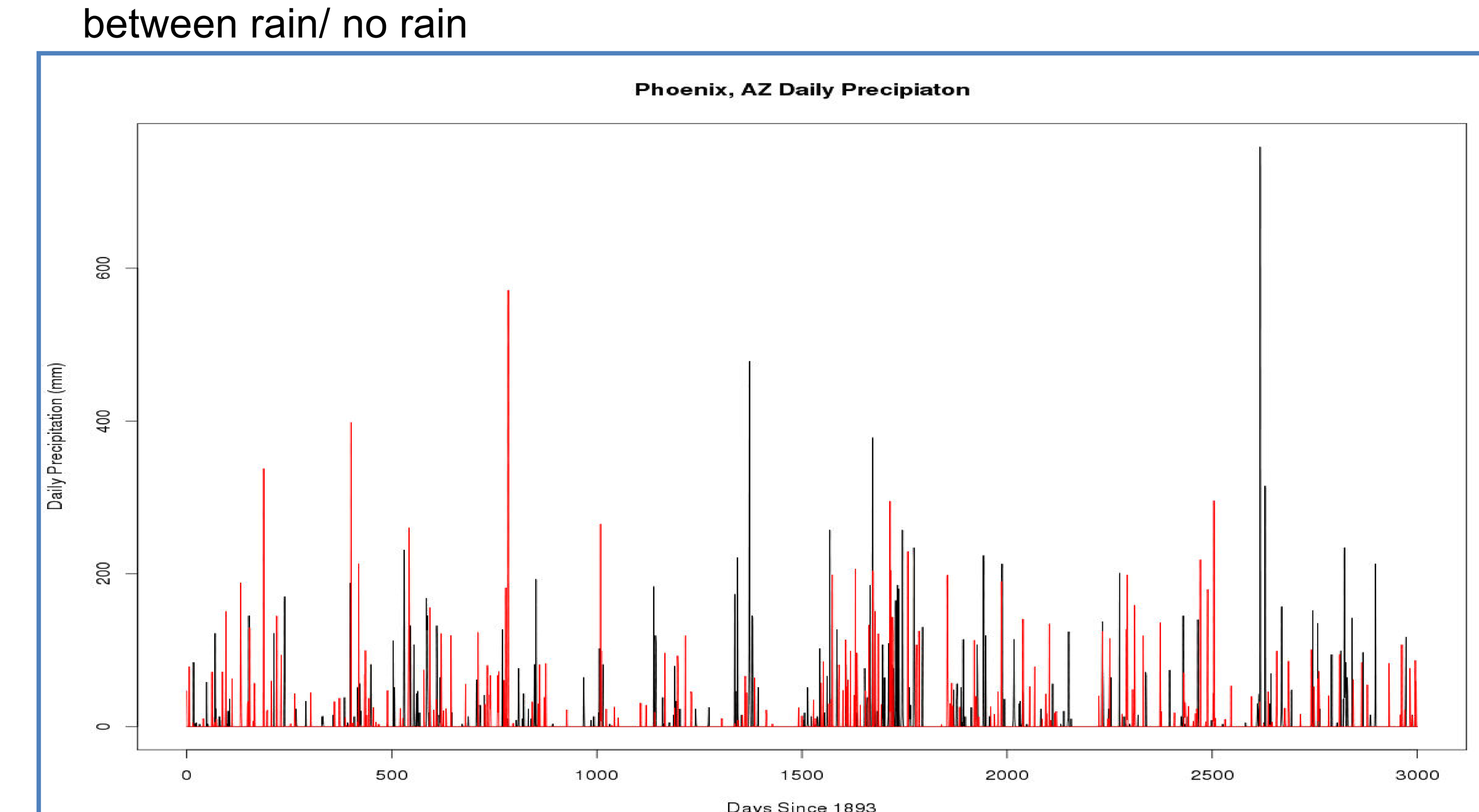

Figure 4 – Phoenix, AZ since 1893
The precipitation data is in black, and a Weibull simulation in red.

Weibull Discussion

- Weibull model with Markov transitions accounts for zeros and extremes
- Seasonal models account for variability in distribution of positive data
- Future: 1) Flexible model dependence in amount of rain
  2) Interpret model fit

References

- Zheng.T., et al., Generalized ARMA models with martingale difference errors. Journal of Econometrics (2015). http://dx.doi.org/10.1016/j.jeconom.2015.03.040
- Wilks, D.S. "Interannual Variability and Extreme-value Characteristics of Several Stochastic Daily Precipitation Models." Interannual Variability and Extreme-value Characteristics of Several Stochastic Daily Precipitation Models. Agricultural and Forest Meteorology, 12 Mar. 1999. Web. 21 Mar. 2016.

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